NATURAL HARMONIC SYSTEMS

The whole tone scale, which is used in all forms of music, is a series of eight tones that occur between fundamental frequencies that are known as octaves. If "f_o" is the fundamental frequency for a series of octave frequencies, each succeeding frequency (f) is defined by the binary series of numbers times the fundamental:
$f = f_o(2,4,8,16,...,2^n)$.
In the series equation, "n" is equal to the whole number series.

There is one whole tone scale between a each pair of consecutive fundamental frequencies in the octave series, and there are eight separate intervals in each scale. If the fundamental frequency is the first tone, then the next one is known as the second tone, or "second" for short. The next tone is known as the third, the next the fourth, the next the fifth, and so on up to the eighth, which is the next octave tone.

The series of eight tone intervals is usually expressed in terms of the frequency of its fundamental tone. If the frequency of the fundamental tone is defined by the number "1," the frequency of each of the scale's eight intervals can be expressed as "1" plus some fractional value less than "1." The fractional values are: (1/8, 1/4, 1/3, 1/2, 2/3, 7/8). When these are converted into decimal numbers and added to the fundamental value (1.00), the result is the eight tones of the whole tone scale:

INTERVAL		FRACTION + 1
Fundamental	-	1.000
Second	-	1.125
Third	-	1.250
Fourth	-	1.333
Fifth	-	1.500
Sixth	-	1.666
Seventh	-	1.875

Whole Tone Intervals as Number Series

It has been seen that the series of octaves is defined by the binary number series, which goes on indefinitely. In a similar manner, different mathematical series can be used to define each of the eight intervals in the whole tone scale. These intervals, and their number series, are listed below:

Octaves: (2,4,8,...)
Seconds: (1.125, 2.25, 4.5, 9,...)
Thirds: (1.25, 2.5, 5,...)
Fourths: (1.333, 2.666, 5.333, 11.666,...)
Fifths: (1.5, 3, 6,...)

Sixths: (1.666, 3.333, 6.666,...)
Sevenths: (1.875, 3.75, 7.50, 15.00,...)

MUSICAL SCALES AND QUANTUM THEORY

The following will show that the natural laws that are used to define the frequencies of the most often used scale in music, the diatonic scale, are the same laws that define the frequencies that quantum theory has defined for the emission of radiation from the Hydrogen atom.

The musical scale that has been used for centuries in the Western countries is the diatonic or "12 tone" scale. The tones or notes in this scale are defined by the series of values that are listed in Fig. 1.

The ratios of the diatonic scale were chosen as part of a subjective process that occurs inside the minds of the many people who listened to Western music in its early, formative days during the Renaissance period of European history. When people listen to music, something inside their head tells them whether it is pleasing or not. When people heard music that was composed to be played on the diatonic scale, they liked it. Somehow, it was pleasing to their ears and mind. Soon this scale replaced the other scales that were in use, and became the standard scale of Western music.

The diatonic scale was not the first scale developed in Europe. Before it there were other scales, each being defined by a mathematical series that varied slightly from the one used to define the notes of the diatonic scale. Besides Western music, there is the Eastern music of China, Japan, India, and many smaller countries. Many of these cultures use scales that are not diatonic. The scale used in India, for instance, has 14 intervals instead of 12. When played on sitars and other instruments, this music sounded unfamiliar at best, and strange at worst, to anyone who has been listening to Western music all their lives.

The basis of the diatonic scale are its eight whole tones, often referred to as: "do, re, mi, fa, so, la, ti, do." The music profession defines these eight tones in terms that are more precise and mathematical but less romantic, with the eight tones being described as "first, second, third, etc."

The first sequence of eight whole tone intervals ends with the first octave, whose frequency is exactly twice that of the fundamental that the sequence began with. If the fundamental has a value of one (1), then the first octave has a value of two (2), the second octave a value of four (4), and etc. The value of each succeeding octave is double the value of the preceding octave. The series of octaves is defined by the binary number series:

$$1,2,4,8,...,2^n, \quad \text{where:} \quad n = 1,2,3,...$$

The diatonic scale also has half tone intervals that occur between the whole tone intervals. There are seven spaces between the scale's eight whole tone intervals. If all of these were occupied (or defined) a half tone, you would have a 14 tone scale. This scale is used in India and some other eastern countries. But the diatonic scale of the West skips two half tone intervals, and reduces the total number of half tones from seven to five.

In the musical notation of Western music, all of the half tone intervals except one are augmentations of the whole tone interval that precedes them in the scale. The only one that is not referred to as being augmented (aug.) is the dominant (dom.) seventh, which is the half tone below the major (maj.) seventh, which is a whole tone.

The Quantum Spectrum of Hydrogen

Light is a frequency of electromagnetic energy. There is also a particle of light energy, a photon, that is one complete frequency cycle for an electromagnetic wave. The electromagnetic frequencies for Hydrogen are defined by a set of mathematical equations, as defined by the following equation:

$$N = [m - (1/n^2)]$$

In the equation, "m" and "n" are whole number series. For each series, each value of "m" is held constant, while the "n" series is allowed to range from an initial value of "m + 1" to a final value of infinity. The first three values of "m" define the first three frequencies series for the element Hydrogen: The Lyman Series (m = 1); the Balmer Series (m = 2), the Paschen Series (m = 3). When the resultant of the equation (N) is multiplied by the Rydberg Frequency Constant (R), a frequency series for hydrogen is found. $R = 3.3 \times 10^{15}$ Hz. The frequencies and wavelengths of the Lyman Series, the only series that has musical qualities, are listed in Figure 2.

Musical Intervals in Hydrogen

The concept of the "information theory of matter" maintains that there is information in the form and structure of atoms that is a portion of "natural law." Natural law in turn can be defined as something that describes the form, fucntioning, and structure of natural systems. Living things would be one form of a natural system.

An example of natural law is the diatonic scale that is used in all forms of music throughout the world. The frequencies of the notes that make up this scale are defined by a series of numerical values that are approximately based upon the twelfth root of the number 2, or in mathematical terminology:

$$f = f_0(2)^{1/12} = 1.05946...$$

Each note in the diatonic scale is determined by multiplying the previous note by this value. The diatonic scale did not develop by happenstance or coincidence. Music has been around for a long time. People have played musical instruments all throughout history. Murals and wall paintings from ancient Egypt show the playing of string and wind instruments.

Having listened to and composed music since antiquity, people have evolved musical tastes as to what constitutes music that is pleasing to the ear. The diatonic scale is based upon this subjective assessment. The mathematical intervals of this scale were not written down by accident or by a scientist or engineer who decided what scale would work best when applied to the design of musical instruments. The scale evolved as a natural consequence of listening to music, and as such is a part of natural law.

In the musical history of Western Europe, there was another scale that was used before the diatonic scale. This was the "true ratio" scale, which became known as the "untempered" scale becasue its intervals were not entirely pleasing or satisfactory to the musical tastes of the people who used it.

After many years of musical experimentation, the untempered scale eventually gave way to the better sounding "well-tempered" scale, which became known as the diatonic scale. The suffix "di" is a shortened version of a Latin term that describes this scale as having 12 tones.

The mathematical ratios of the untempered scale are based upon powers of two instead of the twelfth root of two, or:

$$f = f_0(2)^n \quad \text{where: } n = 0,1,2,3,...,n$$

Mathematicians and computer programmers will recognise this as the binary formula that is the basis for all computer operations. This formula describes the intervals and the octaves in the untempered scale. This scale was divided into seven whole tone intervals (do, re, me, fa, so, la, te, do), the eighth note being the octave and identical to the first.

But a problem arose when the attempt was made to subdivide the whole tone scale into intervals that were between the successive whole tones. The formula only allowed for 15 half tones (the sixteenth is the octave), some of which were most unpleasing to listen to. Through trial and error it was discovered that only a total of five half tones was neceesary to produce a scale that was pleasing to listen to. These 5 half tones were added to the 7 original whole tines for the completed scale of 12 tones. The problem that now was how to define the intervals in this new scale. After a few years of investigation, the twelfth root of 2 formula solved this problem and the modern musical system was established.

Oddly enough, the untempered scale has reappeared in the modern scientific technology of acoustics and ultrasonics. This scale defines the harmonics of a resonating cavity. When a hifi speaker is placed in its enclosure, it will naturally produce a seris of overtones and undertones that are defined by the binary series of numbers. Some of these tones, however, are not compatible which those of the diatonic scale, and the speaker manufacturer must have its technicians tune them out of each enclosure so that they will not produce dissonance when music is player through the speakers.

In order to determine the musical properties of the Lyman Series, the series must be put into the same mathematical as one octave in the musical scale. The Lyman Series ends with the numerical value of unity (1.000). If this is taken as the first octave of the Lyman scale, the fundamental would be one-half of this value (.500). To make the math of the Lyman Series (Fig. 2) equivalent to the math of the diatonic scale (Fig. 1), double the Lyman values. This gives the fundamental of the Lyman scale the value of unity (1.000), and the octave twice a value of "2.000." Now double each of the Lyman ratios under "N" in Fig. 2, and compare them with the diatonic harmonic ratios in Fig. 1.

Figure 3 compares the Lyman and diatonic ratios to each other. The first Lyman value (n = 2) is the harmonic fifth of the diatonic scale. The second Lyman value (n = 3) is the dominant (dom) seventh of the scale, and the third (n = 4) is the major (maj) seventh of the scale. An infinite series of augmented sevenths (also written as "7^{+}") starts at: n = 5. Although this series has not been given a musical notation, the "nth" value in the series could be written as "$7^{++++\cdots}$"

The final value of the hydrogen scale is defined by the "nth" value of infinity (∞). This is the first octave of the hydrogen scale. Practically speaking, there is no such thing as an infinite number of frequencies for hydrogen (or any other element), as this would indicate the presence of an infinite amount of electromagnetic energy for a single atom, something that is impossible. A numerical series can converge to an infinite number, but it can never actually attain it, so the final octave note in the diatonic hydrogen scale must have a frequency that is "Δf" less than the octave.

It can be seen that the Lyman Series for Hydrogen has several values that coincide with those of the diatonic scale of Western music. This seems to suggest that the natural law that governs the diatonic scale of pleasant musical sounds is similar to the one that governs the absorption and emission of light from Hydrogen. There is not extant any physical force, effect, or phenomenon that could be used to predict this coincidence. Perhaps someday there will be.

CHARTS

RATIO	INTERVAL NAME	INTERVAL
1.00	fundamental	whole
1.055	aug. first (1+)	half
1.125	second	whole
1.20139	aug. second (2+)	half
1.25	third	whole
1.33..	fourth	whole
1.3958	aug. fourth (4+)	half
1.50	fifth	whole
1.61806	aug. fifth (5+)	half
1.66..	sixth	whole
1.77..	dom. seventh (-7)	half
1.875	maj. seventh	whole
2.00	first octave	whole

Chart 1: Fractional Values of the Diatonic Scale

n	N	FREQ	WAVELENGTH
2	.7500	2.467×10^{15}	1216
3	.8888..	2.924 "	1026
4	.9375	3.083 "	973
5	.9600	3.157 "	950
6	.9722	3.197 "	938
7	.9796	3.222 "	931
8	.9844	3.237 "	926
9	.9876	3.248 "	924
50	.9996	3.287 "	912.5
.....	.....	.....	.....
∞	1.0000	3.289 "	911.75

Chart 2: The Lyman Series for hydrogen

Lyman	diatonic	diatonic

number(n)	ratio (2N)	interval
2	1.500	5
3	1.77..	-7
4	1.875	7
5	1.920	7+
6	1.944	7++
7	....	7+++
....		
n	2.000	8th (octave)

Chart 3: Comparison of Lyman and Diatonic Ratios

PYTHAGORAS AND MUSIC

The whole tone scale is the basis for all modern musical compositions. This scale has seven different notes or intervals. The eighth note in the scale is the first note of the next octave, which then has seven more notes until the next octave is reached.

The mathematics of the whole or eight tone scale were defined by the Greek Mathematician Pythagoras, who assigned definite fractional values to each successive note in the scale. These values are given in one of the charts that accompany this section.

The interval between the fundamental frequency of the whole tone scale and the fifth note in the scale is known as the "harmonic fifth". The ratio of frequencies for the harmonic fifth is "1.5". To find the harmonic fifth of any scale, its fundamental is multiplied by this value. Other numerical values define other harmonic intervals, as shown below.

The octave interval is the basic mathematical structure for all musical instruments and compositions. The octave interval is defined by the binary number series: $n = 1,2,4,8,...,2^n$. The series of octaves begins with a fundamental that has twice the frequency of the previous fundamental. The frequency of the first or fundamental note in each octave is likewise twice that of the fundamental in the previous octave. The octave series is a series of notes that all sound alike when they are played on a musical instrument, the only difference between them being that they have higher or lower frequencies that are defined by the binary series.

In addition to the octave series, all of the intervals in the whole tone scale can be defined by a numerical value. These intervals are listed in the chart, beginning with the harmonic second, which is the interval between the fundamental and the second note in the scale.

The Pythagorean Comma

The Pythagorean Comma is a mathematical artifact that can be defined as the difference between the mathematical ratios of the octave series and any other series of intervals, such as the harmonic fifths, fourths, etc.

The "Comma" for the series of fifths and octaves is defined as follows: If the first interval of the harmonic fifth is added to the second harmonic fifth, and so on until the series of fifths is equal to the series of octaves, the convergence between the two series is attained with twelve harmonic fifths and seven octaves. That is, twelve successive harmonic fifth intervals are equal to seven successive octave intervals in the whole tone scale of eight tones.

All musical intervals convergence to the octave series after some number of repetitions, however, the actual frequencies of the individual series do not converge to a common number. The inexactness or "divergence" between the series of fifths and the octave series is defined by the following number ratio: 129.7463/128 = 1.0136. The top number is the frequency of the "12th" harmonic fifth and the bottom is the frequency of the "7th" octave. In both cases, the starting frequency was given a value of "1".

The following chart shows the Pythagorean Comma calculations for the for all of the intervals in the whole tone scale. The equation in the second column is set up to calculate the ratio of the series of harmonic intervals (fifths, fourths etc.) to the series of octave intervals for the "point of convergence" of the two series.

The number in the right hand column is the divergence between the two series (octave and harmonic interval). This is the Pythagorean Comma. Theoretically, this number should always be "1" because for any musical instrument, the interval and the octave are the same note, but the inexactness of the mathematical ratios in the whole tone scale allow for the predicted divergence.

INTERVAL	EQUATION		CALCULATION		COMMA
Fifths:	$[(1.5)^{12}/ 2^{7}]$	=	129.7463/128	=	1.0136
Seconds:	$[(1.125)^{6} / 2]$	=	2.0273/2	=	1.0136
Fourths:	$[2^{6} / (1.33)^{12}]$	=	63.0799/64	=	.9856
Thirds:	$[(2^{2} / (2.50)^{3}]$	=	3.9063/4	=	.9766
Sixths:	$[2^{3} / (1.66)^{4}]$	=	15.4313/16	=	.9645

Sevenths: $[2^{11} / (1.875)^{12})$ = 1888.0577/2048 = .9219

The following chart shows the ratio of harmonic fifths for all of the notes (beginning with C) in the twelve tone scale. The deviation from the perfect fifth, the Pythagorean Comma, is the difference between the perfect fifth, which is "1.5" times the fundamental, and the "imperfect" fifths, which are either slightly greater than (or less than) "1.500." Note that out of twelve intervals, six are perfect fifths and six are not.

FIFTH	RATIO
C - G	1.500
C# - G#	1.5329
D - A	1.48
D# - A#	1.4797
E - B	1.500
F - C	1.500
F# - C#	1.5124
G - D	1.500
G# - D#	1.485
A - E	1.500
A# - F	1.500
B - F#	1.49